Functions – More about the zeros of polynomials

Given the quadratic polynomial, $x^2 - 3x - 10$, you can find the zeros of the polynomial by factoring.

$x^2 - 3x - 10 = 0$ ⟫ $(x-5)(x+2) = 0$ ⟫ so x = 5 and x = –2

So since, $x^2 - 3x - 10 = (x-5)(x+2)$ simple division yields ⟫ $\frac{x^2-3x-10}{(x-5)} = (x+2)$

$$
\begin{array}{r|l}
 & x - \ \ 2 \\
x - 5 & x^2 - 3x - 10 \\
- & x^2 - 5x \\
- & \quad 2x - 10 \\
 & \quad 2x - 10 \\
 & \qquad 0
\end{array}
$$

If you divide a polynomial by one of its factors, the

Remainder **will be 0**.

Notice the quotient (answer part) is the remaining factor.

So suppose you wanted to find the zeros of the polynomial: $x^3 - 5x^2 - 2x + 24$.

This isn't the easiest thing to factor.

But suppose you knew that one of the zeros was 3.

You can use this fact and long division to simplify the problem.

If 3 is a zero, then that means that $x^3 - 5x^2 - 2x + 24 = 0 \; when \; x = 3$

The zero $x = 3$ corresponds to the factor $(x-3)$.

$$
\begin{array}{r|l}
 & x^2 - 2x - \ \ 8 \\
x - 3 & x^3 - 5x^2 - 2x + 24 \\
- & x^3 - 3x^2 \\
- & \quad -2x^2 - 2x \\
 & \quad -2x + \ 6x \\
- & \qquad -8x + \ 24 \\
 & \qquad -8x + \ 24 \\
 & \qquad\qquad 0
\end{array}
$$

This means that $\frac{x^3-5x^2-2x+24}{x-3} = x^2 - 2x - 8$ OR

$x^3 - 5x^2 - 2x + 24 = (x-3)(x^2 - 2x - 8)$

To find the remaining zeros, you only have to factor the remaining quadratic factor.

Factoring $x^2 - 2x - 8 = (x-4)(x+2)$

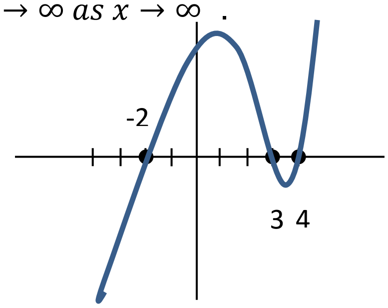
Therefore

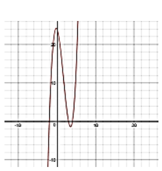
$x^3 - 5x^2 - 2x + 24 = (x-3)(x-4)(x+2)$

Now that you know the zeros of the polynomials, you can sketch it.

Type equation here.ince the polynomial has an odd degree and a positive leading coefficient, the end behavior is $y \to -\infty \; as \; x \to -\infty \;\; and \;\; y \to \infty \; as \; x \to \infty$.

So the polynomial looks something like:





It actually looks like:

What if you divided $x^3 - 5x^2 - 2x + 24$, by some factor that ISN'T a zero, what happens?

Try $\frac{x^3-5x^2-2x+24}{x-5}$

$$
\begin{array}{r|l}
 & x^2 \qquad - 2 \\
x - 5 & x^3 - 5x^2 - 2x + 24 \\
- & x^3 - 5x^2 \\ \hline
- & 0 \quad - 2x + 24 \\
 & \quad -2x + 10 \\ \hline
 & \qquad\qquad 14
\end{array}
$$

The remainer is 14.

For a factor of (x – 5), x = 5.

What happens when you plug 5 into the original polynomial?

$f(5) = (5)^3 - 5(5)^2 - 2(5) + 24 = 125 - 125 - 10 + 24 = 14$ which was the same as the remainder.

Remainder Theorem (also known as little Bezout's theorem) states:

If you divide a polynomial, $f(x)$, by a factor $(x - a)$, for some number a, the remainder of $\frac{f(x)}{x-a} = f(a)$, which is THE SAME value as plugging a into the original polynomial, $f(x)$.

Ex. Given the polynomial $f(x) = x^2 - 5x + 6$, is:

a) 4 a zero?

$f(4) = 4^2 - 5(4) + 6 = 16 - 20 + 6 = 2$

So NO.

Why?

$$
\begin{array}{r|l}
 & x - 1 \\
x - 4 & x^2 - 5x + 6 \\
- & x^2 - 4x \\ \hline
- & \quad -x + 6 \\
 & \quad -x + 4 \\ \hline
 & \qquad 2
\end{array}
$$

b) 3 a zero?

$f(3) = 3^2 - 5(3) + 6 = 9 - 15 + 6 = 0$

So YES.

Why?

$$
\begin{array}{r|l}
 & x - 2 \\
x - 3 & x^2 - 5x + 6 \\
- & x^2 - 3x \\ \hline
- & \quad -2x + 6 \\
 & \quad -2x + 6 \\ \hline
 & \qquad 0
\end{array}
$$